

Answer all questions. Write your answers in the spaces provided.

1. A sequence a_1, a_2, a_3, \dots is defined by

$$\begin{aligned}a_{n+1} &= 4 - a_n \\ a_1 &= 3\end{aligned}$$

Find the value of

(a) (i) a_2

(ii) a_{107}

(2)

(b) $\sum_{n=1}^{200} (2a_n - 1)$

(2)

2. A circle C has equation

$$x^2 + y^2 + 4x - 10y - 21 = 0$$

Find

- (a) (i) the coordinates of the centre of C ,

- (ii) the exact value of the radius of C .

(3)

The point $P(5, 4)$ lies on C .

- (b) Find the equation of the tangent to C at P , writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(4)

3. (i) Use algebra to prove that for all real values of x

$$(x - 4)^2 \geq 2x - 9 \quad (3)$$

- (ii) Show that the following statement is untrue.

$$2^n + 1 \text{ is a prime number for all values of } n, n \in \mathbb{N} \quad (1)$$

4. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\left(2 - \frac{1}{4}x\right)^6$$

(4)

- (b) Given that x is small, so terms in x^4 and higher powers of x may be ignored, show

$$\left(2 - \frac{1}{4}x\right)^6 + \left(2 + \frac{1}{4}x\right)^6 = a + bx^2$$

where a and b are constants to be found.

(3)

5. A company makes a particular type of watch.

The annual profit made by the company from sales of these watches is modelled by the equation

$$P = 12x - x^{\frac{3}{2}} - 120$$

where P is the annual profit measured in thousands of pounds and $\pounds x$ is the selling price of the watch.

According to this model,

- (a) find, using calculus, the maximum possible annual profit.

(6)

- (b) Justify, also using calculus, that the profit you have found is a maximum.

(2)

6. $f(x) = kx^3 - 15x^2 - 32x - 12$ where k is a constant

Given $(x - 3)$ is a factor of $f(x)$,

- (a) show that $k = 9$

(2)

- (b) Using algebra and showing each step of your working, fully factorise $f(x)$.

(4)

- (c) Solve, for $0 \leq \theta < 360^\circ$, the equation

$$9 \cos^3 \theta - 15 \cos^2 \theta - 32 \cos \theta - 12 = 0$$

giving your answers to one decimal place.

(2)

7. Kim starts working for a company.

- In year 1 her annual salary will be £16 200
- In year 10 her annual salary is predicted to be £31 500

Model *A* assumes that her annual salary will increase by the same amount each year.

- (a) According to model *A*, determine Kim's annual salary in year 2. (3)

Model *B* assumes that her annual salary will increase by the same percentage each year.

- (b) According to model *B*, determine Kim's annual salary in year 2. Give your answer to the nearest £10 (3)
- (c) Calculate, according to the two models, the difference between the total amounts that Kim is predicted to earn from year 1 to year 10 inclusive. Give your answer to the nearest £10 (3)

8. (i) Find the exact solution of the equation

$$8^{2x+1} = 6$$

giving your answer in the form $a + b \log_2 3$, where a and b are constants to be found. (4)

(ii) Using the laws of logarithms, solve

$$\log_5(7 - 2y) = 2 \log_5(y + 1) - 1 \quad (5)$$

9. (a) Show that the equation

$$\cos \theta - 1 = 4 \sin \theta \tan \theta$$

can be written in the form

$$5 \cos^2 \theta - \cos \theta - 4 = 0 \quad (4)$$

(b) Hence solve, for $0 \leq x < \frac{\pi}{2}$

$$\cos 2x - 1 = 4 \sin 2x \tan 2x$$

giving your answers, where appropriate, to 2 decimal places. (4)

10.

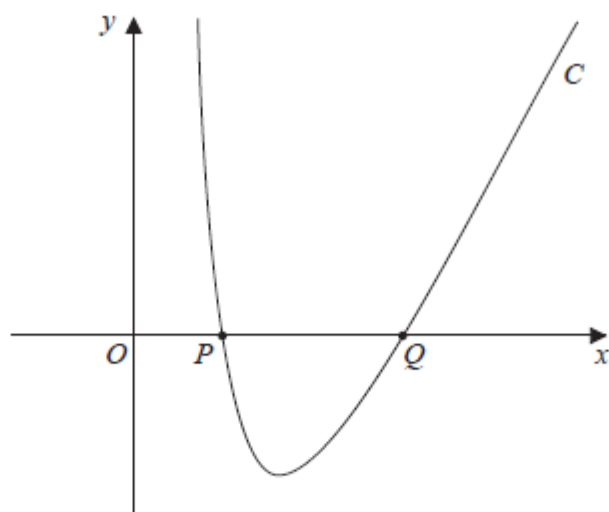


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = f(x)$ where

$$f(x) = \frac{36}{x^2} + 2x - 13 \quad x > 0$$

Using calculus,

- (a) find the range of values of x for which $f(x)$ is increasing,

(4)

- (b) show that $\int_2^9 \left(\frac{36}{x^2} + 2x - 13 \right) dx = 0$

(4)

The point $P(2, 0)$ and the point $Q(6, 0)$ lie on C .

Given $\int_2^6 \left(\frac{36}{x^2} + 2x - 13 \right) dx = -8$

- (c) (i) state the value of $\int_6^9 \left(\frac{36}{x^2} + 2x - 13 \right) dx$

- (ii) find the value of the constant k such that $\int_2^6 \left(\frac{36}{x^2} + 2x + k \right) dx = 0$

(3)